Type Checking Algorithm for $\lambda \rightarrow$

\[
TC(\Gamma, \text{true}) = \text{Bool}
\]
\[
TC(\Gamma, \text{false}) = \text{Bool}
\]
\[
TC(\Gamma, 0) = \text{Nat}
\]
\[
TC(\Gamma, x) = \Gamma(x)
\]
\[
TC(\Gamma, \text{pred } t) = \text{Nat} \text{ if } TC(\Gamma, t) = \text{Nat}
\]
\[
TC(\Gamma, \text{succ } t) = \text{Nat} \text{ if } TC(\Gamma, t) = \text{Nat}
\]
\[
TC(\Gamma, \text{iszero } t) = \text{Bool} \text{ if } TC(\Gamma, t) = \text{Nat}
\]
\[
TC(\Gamma, \text{if } t_1 \text{ then } t_2 \text{ else } t_3) = T \text{ if } TC(\Gamma, t_1) = \text{Bool} \text{ and } TC(\Gamma, t_2) = T \text{ and } TC(\Gamma, t_3) = T
\]
\[
TC(\Gamma, t_1 t_2) = T \text{ if } TC(\Gamma, t_1) = T' \rightarrow T \text{ and } TC(\Gamma, t_2) = T
\]
\[
TC(\Gamma, \lambda x : T'. t) = T' \rightarrow T \text{ if } TC((\Gamma, x : T'), t) = T
\]

**Theorem 0.1 (Soundness of Type Checking).** If $TC(\Gamma, t) = T$ then the judgement $\Gamma \vdash t : T$ is derivable.

**Proof.** By structural induction and case analysis on $t$.

**Theorem 0.2 (Completeness of Type Checking).** If $\Gamma \vdash t : T$ is derivable then $TC(\Gamma, t) = T$.

**Proof.** By structural induction and case analysis on $t$. 