Study Guide & Scavenger Hunt **November 12, 2007**

These problems were chosen to help prepare for the second midterm exam, scheduled for Friday, November 16, 2007. Note: some of these problems are much harder than the ones that will appear on the exam. However, every correct solution that is turned in before you take the midterm will count as 1 extra credit point added to your midterm exam score, up to a maximum of 20 extra-credit points. *You must show all of your work for full credit. And your solutions must be unique. EXPLAIN, EXPLAIN, EXPLAIN. GOOD LUCK!*

1. Write a short scheme program (from memory, i.e., without copying one from your notes) that computes the factorial of an arbitrary integer n.
2. Write a short scheme program that computes the binomial coefficient $\binom{n}{k}$ for two arbitrary integers n and k. (Try to also do this one from memory.)
3. (♦♦) Niclaus Bernoulli (1687–1759) wrote ten letters each one addressed to a different individual, and then addressed ten envelopes, one for each of the ten recipients. How many different ways can the ten letters be placed in the ten envelopes, such that each envelope contains an incorrect letter?
4. Let the list (`R R R R R R E B B B B B B`) denote the initial state of a twelve-peg puzzle, similar to the linear eight-peg puzzle discussed in class. Recall that red pegs can only move to the right, blue pegs to the left. A peg can either move into the empty hole (E) by a single step, or by jumping over a single peg. The goal state is denoted by the list (`B B B B B B E R R R R R R`).
   (*a*) Without solving the puzzle, determine the length of the solution path, that is how many moves are required to solve the puzzle. A single jump and a single step are each considered to be *one* move. Explain how you obtained this answer.
   (*b*) Does a solution exist? If so, please describe one solution path of the puzzle by listing the sequence of states using list notation.
5. Write a short program in scheme that swaps the first and last symbol of a list that contains only a pair of symbols. Thus `(pair-swap '(a b))` should return `(b a)`.
6. Write a longer program in scheme that swaps the first and last symbol of a list that contains two *or more* symbols. Thus `(end-swap '(a b c d e f g))` should return `(g b c d e f a)`.
7. Please evaluate the following scheme expressions:
   (a) `(null? 0)`
   (b) `(cons 'vampire (cons 'professor '(ghost)))`
   (c) `(car (car (cdr '(car (watch the quote!)))))`
   (d) `(car '((boo!)))`
   (e) `(list-ref '(a b c d e f) 1)`

8. Consider the function peggy:

```
(define peggy
  (lambda (s)
    (cond ((null? s) #f)
          ((null? (cdr s)) #f)
          ((null? (cddr s)) #f)
          ((and (eq? 'R (car s)) (eq? 'H (caddr s)))
           (cons 'H (cons (cadr s) (cons 'R (cdddr s)))))
          (else (cons (car s) (peggy (cdr s)))))))
```

What does peggy do? (Hint: Evaluate `(peggy '(R R H B B))` and `(peggy '(B B H R R))`).

9. Please write a scheme program `(digits` n`)` that returns a list of integers that runs from 1 to n. Thus `(digits 8)` should return `(1 2 3 4 5 6 7 8)`.

10. Please express the decimal number 97 in base two.

11. In a game of ordinary nim the heaps have sizes $(3\ 5\ 11\ 17\ 24\ 27)$. Please compute the parity sum of these six numbers.

12. List every initial move that balances the heaps in the previous problem.

13. In a game of restricted nim, each player is allowed to remove either 1, 2, or 4 tokens from a single heap. Players alternate turns, and the player who removes the last token wins. Suppose the game begins with two heaps in the position $(3\ 8)$. What should the first player do to ensure victory?

14. In some variants of mancala, players are allowed to redistribute their 24 tokens over their six pits in any manner they please before the first move is made. In how many different states can this game begin?

15. In how many different ways can 64 seeds be distributed over the 32 pits in a bao board, so that each pit contains at least one seed.

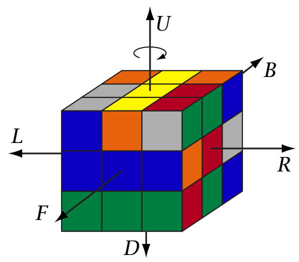
16. The game of Go is played on a nineteen-by-nineteen square grid. During each turn a player may either pass, or place a single stone of his or her color on an unoccupied intersection of the grid. If by placing that stone the player is able to completely surround a group of the opposing player's stones in the horizontal and vertical directions, then the opposing player's stones are removed from the board, and count as points. There are other interesting rules to the game, which you can discover on the internet. For this problem though, I would like you to estimate the maximum number of postions (or states) that can be realized in the game of Go.

17. Checkers is played on the 32 dark squares of an 8-by-8 board. Twelve pieces are red, and twelve are black. During the course of the game, some pieces are captured, some are promoted to kings, and some remain as they are. Please *estimate* the number of states in the game of checkers. Note that the exact answer might not be known, so please try your best by applying the counting tricks that we have learned in class.

18. What is the maximum number of pieces of pizza that can be created by dividing a pizza with 6 straight cuts, without moving the slices between cuts. Note that one cut divides a pizza into two pieces, two cuts divides it into at most four, but three cuts divides it into at most seven.

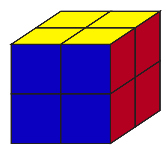
19. Consider a $3 \times 3 \times 3$ Rubik's cube. Let U denote the move that rotates the upper (yellow) face by 90° in the clockwise direction (as shown). Similarly, let F, R, L, B, and D denote 90° rotations in the clockwise direction of the remaining five faces. We use the shorthand $R^2 = RR$, and $R' = R^3$.

    Suppose a cube is scrambled according to the following sequence of operations: $R^2BD'F^2U$. Construct a sequence of operations that unscrambles the cube.

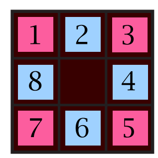


20. How many different states can be reached on $3 \times 3 \times 3$ Rubik's cube (without breaking the cube apart into pieces)? Show your work!

21. Consider a $2 \times 2 \times 2$ Rubik's cube. How many different configurations can be reached by a legal sequence of moves? Explain.



22. Consider the 8-puzzle, an 8 tile simplification of Sam Loyd's 15-puzzle. The goal state of the puzzle is shown below, note that the blank is in the middle.How many different configurations are reachable by sliding a sequence of tiles in the frame?



23. In a game of *nim misère* the player who is forced to remove the last stone loses. For each of the following states (or positions), please identify every optimal move for the next player. (Please also indicate if no optimal moves exist).

    (a) `(5 6 7 8 9)`

    (b) `(4 6 8 10)`

    (c) `(1 1 1 4)`

24. In Whytoff's version of nim, the tokens are initially arranged into two heaps. During each turn a player may remove an arbitrary number of tokens from either heap, *or* the same number of tokens from both heaps. Please indicate a winning move (if one exists) for the state *(4 5)*. (Hint: Whytoff's nim is isomorphic to Whytoff's Queen Game, which we studied in class.)

25. What does the following scheme function compute?

```
(define mystery
  (lambda (s)   ; s should be a list of non-negative integers.
    (letrec ((aux
              (lambda (n)
                 (cond ((memq n s) (aux (+ 1 n)))
                       (else n)))))
      (aux 0))))
```

Hint: Evaluate `(mystery '())`, `(mystery '(0))`, `(mystery '(0 1))`, etc.